PERTH MODERN SCHOOL
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Independent Public School

Course   Specialist   Year 11   Test 2 2022

Student name: _____   Teacher name: _____

Task type: **Response**

Time allowed for this task: 40 mins

Number of questions: 5

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **40 marks**

Task weighting: **10 %**

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 **(6 marks)**

Consider the three vectors $a = 2i - 3j$, $b = 3i + mj$ and $c = i - 2j$, where $m \in \mathbb{R}$.

(a) Find the value(s) of m for which $|b| = 5\sqrt{3}$. (2 marks)

(b) Calculate $a - 2b$, leave your answer in terms of m. (2 marks)

(c) Find the value(s) of m such that $a - 2b$ is perpendicular to c. (2 marks)

Question 2 **(9 marks)**

(a)   Determine the number of integers between 1 and 450 inclusive that are divisible by 2 or 7.

(4 marks)

(b)   A selection of three athletes is to be formed from 5 Australian, 7 American and 6 European athletes. Determine the number of different selections of three athletes if

(i)   there are no restrictions.   (1 mark)

(ii)   the selection must have one athlete from each continent.   (2 marks)

(iii)   the selection must have at least two athletes from Australia.   (2 marks)

Question 3 **(8 marks)**

Relative to the origin O, Points A and B have position vectors $a = 4i - 3j$ and $b = -2i + j$, respectively.

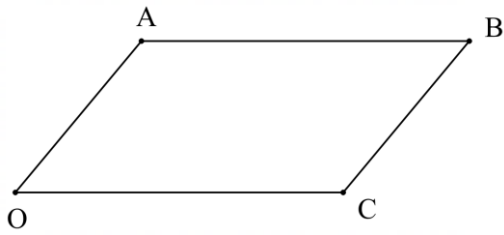   a) Determine the exact unit vector $\hat{c}$, where $\mathbf{c} = \overrightarrow{AB}$.   (3 marks)

    b) Vector $\mathbf{d}$ has magnitude $5\sqrt{13}$, is parallel to $\mathbf{c}$ and in the opposite direction. Determine $\mathbf{d}$.

                                                        (2 marks)

    c) Determine the vector projection of a in the direction of b.   (3 marks)

Question 4 (8 marks)

Let $OABC$ be a parallelogram, with sides $\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OC} = \mathbf{c}$

(a) Determine the vectors $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. (2 marks)

(b) Use a vector method, to prove that the sum of the squares of the lengths of the diagonals of $OABC$, is equal to the sum of the squares of the lengths of all four sides of $OABC$. (3 marks)

(c) What can be said about the parallelogram if the diagonals are perpendicular? (3 marks)

Question 5 **(9 marks)**

a) A dry cleaner selects shirts at random from a laundry bag. The laundry bag is known to contain 2 white shirts, 3 green shirts, 5 blue shirts, 8 yellow shirts and 10 red shirts.

Use the pigeonhole principle to determine how many shirts have to be drawn to guarantee that the dry cleaner has at least three of one colour. Justify your answer. (3 marks)

b) In Western Australia all motorcycle licence plates begin with a 1, followed by three letters and then three numbers, to create the 7-character registration plate standard.
Example: 1AAA.000

(i) How many different licence plates are possible? (2 marks)

(ii) In March 2020, the general plates had reached the "1H" sequence. How many different licence plates are possible, with this sequence? (1 mark)

(iii) How many different plates contain only odd digits and exactly two vowels, with no repeated letters or digits? (3 marks)

Extra Working Space

Question_____